Reg. No. | | | | | | | | | |

B.E. / B.TECH. DEGREE EXAMINATION, MAY 2017

FIRST SEMESTER

MA16151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2016)

Q. Code: 290061

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** Questions

Part A (10 x 2 = 20 Marks)

1. Find the eigen values of adj A , where $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$.
2. Find the nature of quadratic form $xy + yz + zx$.
3. Define convergence of a sequence.
4. Test for convergency the series $\sum \frac{n!}{n^n}$.
5. Find the curvature at (-2,0) on $y^2 = x^3 + 8$.
6. Obtain the envelope of $y = mx + \frac{a}{m}$, where m is a parameter.
7. State any two properties of Jacobian.
8. Calculate the stationary value for $x^2 + y^2 + 6x + 12$.
9. Change the order of integration in $\int_0^2 \int_0^x f(x, y) dydx$.
10. Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2z \, dzdydx$.

Part B (5 x 16 = 80 marks)

11\. (a)(i) Identify the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **(8)**

(ii) Using Cayley-Hamilton theorem, obtain A^{-1} for $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$ **(8)**

(OR)

(b) Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. **(16)**

12. (a)(i) Show that $\left\{\frac{n+1}{2n+7}\right\}$ is convergent. **(8)**

(ii) Test the convergence of the series $\sum \frac{1}{\sqrt{1+n^2}}$ **(8)**

(OR)

(b)(i) Using integral test, show that the series $\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \cdots to \infty$ diverges. **(8)**

(ii) Discuss the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \cdots \infty; 0 < x < 1.$ **(8)**

13. (a)(i) Find the radius of curvature for $\sqrt{x} + \sqrt{y} - 1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$. **(6)**

(ii) Obtain the equation of circle of curvature of the rectangular hyperbola $xy = 12$ at the point (3,4). **(10)**

(OR)

(b)(i) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. **(8)**

(ii) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are related by the equation $a^n + b^n = c^n$, c being a constant. **(8)**

14. (a)(i)If $z = u(x, y)$, where $x = e^u \cos v\ ; y = e^u \sin v$,show that **(8)**

$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$.

(ii) Expand $e^x \sin y$ in powers of x and y as far as terms of the third degree. **(8)**

(OR)

(b)(i) Show that $u = xy + yz + zx; v = x^2 + y^2 + z^2; w = x + y + z$ are functionally dependent. **(6)**

(ii) Obtain the volume of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(10)**

15. (a)(i) Evaluate $\iint_R (4 \quad x^2 \quad y^2)dx\,dy$, if the region R is bounded by the straight lines $x = 0; x = 1; y = 0; y = \frac{3}{2}$. **(8)**

(ii) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dxdy$ by changing the order of integration. **(8)**

(OR)

(b)(i) Find the area of the cardioid $r = a(1 + cos\theta)$. **(8)**

(ii) Obtain the volume of the tetrahedron bounded by the co ordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(8)**